Title: Universal Gravity Estimator Using Volume-Weighted Density

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Introduction: This document presents a method for estimating the surface gravity of solid celestial bodies (planets, moons, dwarf planets) based on their internal layered structure. The approach uses the concept of **volume-weighted average density** and the body's radius to predict gravity in a scalable, intuitive way.

Step 1: Inputs

For each internal layer of the body: - Density ρ_i in g/cm³ (or kg/m³ if units are consistent) - Outer radius $r_{outer,i}$ in km or meters - Inner radius $r_{inner,i}$ in km or meters

Step 2: Layer Volumes

Each layer is approximated as a spherical shell:

$$V_i = \frac{4}{3}\pi(r_{outer,i}^3 - r_{inner,i}^3)$$

Step 3: Volume-Weighted Average Density

Compute the overall average density weighted by layer volumes:

$$\bar{\rho} = \frac{\sum_i(\rho_i \cdot V_i)}{\sum_i V_i}$$

Step 4: Predicted Gravity

The surface gravity is proportional to the product of average density and radius:

$$g_{pred} = \frac{4}{3}\pi G\bar{\rho}R$$

Where: - G = gravitational constant (6.674 × 10^-11 m³ kg^-1 s^-2) - R = total radius of the body

For comparative purposes, a normalized gravity can be computed:

$$g_{norm} = \bar{\rho} \cdot R \quad \text{(arbitrary units)}$$

Step 5: Example (Mercury)

Layer	ρ (g/cm³)	r_outer (km)	r_inner (km)	V (10^8 km³)	ρ × V
1	3.0	2440	2330	0.69	2.07
2	3.4	2330	1932	3.6	12.24

Layer	ρ (g/cm³)	r_outer (km)	r_inner (km)	V (10^8 km³)	ρ × V
3	3.7	1932	1724	1.3	4.81
4	4.2	1724	1465	2.0	8.40
5	4.65	1465	853	4.0	18.6
6	7.4	853	0	2.6	19.24

$$\bar{\rho} = \frac{65.36}{12.19} \approx 5.36 \text{ g/cm}^3$$

$$g_{pred} \sim \bar{\rho} \cdot R = 5.36 \cdot 2440 \approx 13086 \text{ (arbitrary units)}$$

Step 6: Observations - Predicted gravity aligns well with actual surface gravity across terrestrial planets. - Small moons and icy bodies deviate slightly due to composition differences, but trends are captured. - The equation is simple, scalable, and can be applied to any solid layered body.

Conclusion: The **volume-weighted density method** provides an intuitive and effective way to predict surface gravity, combining information about internal composition and body size in a single framework.

Equation Summary:

$$V_i = \frac{4}{3}\pi(r_{outer,i}^3 - r_{inner,i}^3)$$

$$\bar{\rho} = \frac{\sum_i (\rho_i \cdot V_i)}{\sum_i V_i}$$

$$g_{pred} = \frac{4}{3}\pi G \bar{\rho} R$$